Program Transformation Seminar - Transforming and solving Linear Programs

Linear Program in standard form

$$\begin{aligned}
\text{maximize} \quad & c_1x_1 + c_2x_2 + \ldots c_nx_n \\
\text{subject to} \quad & a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1 \\
& a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2 \\
& \vdots \\
& a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m \\
& x_1, x_2, \ldots, x_n \geq 0
\end{aligned}$$

This can be rewritten in matrix form as:

$$\begin{aligned}
\text{maximize} \quad & \mathbf{c} \cdot \mathbf{x} \\
\text{subject to} \quad & A\mathbf{x} \leq \mathbf{b} \\
& \mathbf{x} \geq 0
\end{aligned}$$

with $\mathbf{x}, \mathbf{c} \in \mathbb{R}^n, \mathbf{b} \in \mathbb{R}^m, A \in \mathbb{R}^{m \times n}$. Note how this can only handle positive values for x, less than inequality constraints and a maximization of the objective function.

For an introduction on how to transform common linear programs into standard form see [1].

Some examples

Your program should be able to parse, transform into standard form and then solve the following linear programs:

$$\begin{aligned}
\text{maximize} \quad & x_1 + 2x_2 \\
\text{subject to} \quad & x_1 + x_2 \leq 4 \\
& -x_1 + x_2 \leq 1 \\
& -3x_1 + 10x_2 \leq 15 \\
& x_1, x_2 \geq 0
\end{aligned}$$

Optimal solution[2]: $\frac{79}{13}$ at $x_1 = \frac{25}{13}, x_2 = \frac{27}{13}$.

$$\begin{aligned}
\text{maximize} \quad & -2x_1 + 3x_2 - 5x_3 \\
\text{subject to} \quad & 7x_1 - 5x_2 + 6x_3 \leq 10 \\
& -2x_1 + 8x_2 + 4x_3 \leq 3 \\
& 9x_1 - 2x_2 - 5x_3 \leq 4 \\
& x_2, x_3 \geq 0
\end{aligned}$$

[1] https://www.matem.unam.mx/~omar/math340/std-form.html
[2] https://www.cs.toronto.edu/~jepson/csc373/lectures/linearProgIntro_1pp.pdf

$$\begin{aligned} \text{maximize} \quad & x_1 + x_2 + x_3 \\ \text{subject to} \quad & x_1 - x_2 + x_3 = 0 \\ & x_1 + x_3 \leq 5 \\ & x_1 + x_2 \geq 3 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Possible extensions:

- Solve nonlinear optimization problems w. linear bounds