Program Transformation Seminar

Simple Example

$$\begin{aligned} x_0 + 2x_1 &= 0 \\ x_1 + x_2 &= 1 \\ x_0 + x_1 + x_2 &= 0 \end{aligned}$$

The matrix representation of this linear equation system is:

$$A\mathbf{x} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

The determinant of the matrix A is $\det(A) = 2.0$, thus the system is solvable. The matrix is asymmetric, we choose LU to solve the system. The LU decomposition of the matrix A is:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

By backwards and forwards substitution we get the solution:

$$\mathbf{x} = \begin{bmatrix} -1.0 \\ 0.5 \\ 0.5 \end{bmatrix}$$

More involved example

We allow fractions and arithmetic expressions on the factors: Constants to be covered are π, e. Functions to be covered are $\sin, \cos, \tan, \log, \exp, \sqrt{\circ}$

$$\begin{aligned} \frac{1}{1+\pi^2} x_0 + \frac{1}{\sqrt{2}} x_1 &= 1 \\ \frac{\sqrt{2}}{2} x_1 + x_2 &= 0 \end{aligned}$$

The matrix representation of this linear equation system is:

$$A\mathbf{x} = \begin{bmatrix} 0.092 & 0.7071 \\ 0.7071 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The determinant of the matrix A is $\det(A) = -0.408$, thus the system is solvable. The matrix is symmetric, we choose QR decomposition to solve the system. The QR decomposition of the matrix A is:

$$Q = \begin{bmatrix} -0.1290 & -0.9916 \\ -0.9916 & 0.1290 \end{bmatrix} \quad R = \begin{bmatrix} -0.7131 & -1.0829 \\ 0 & -0.5722 \end{bmatrix}$$

By solving $Rx = Q^T b$ we get the solution:

$$\mathbf{x} = \begin{bmatrix} -2.4510 \\ 1.7331 \end{bmatrix}$$

Possible extensions:

- Solve inequality systems
- Solve constrained linear optimization problems in standard form